
**Information technology — Z formal
specification notation — Syntax, type
system and semantics**

*Technologies de l'information — Notation Z pour la spécification formelle —
Syntaxe, système de caractères et sémantique*



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Foreword

ISO (the International Organization for Standardization) and IEC (the International Electrotechnical Commission) form the specialized system for worldwide standardization. National bodies that are members of ISO or IEC participate in the development of International Standards through technical committees established by the respective organization to deal with particular fields of technical activity. ISO and IEC technical committees collaborate in fields of mutual interest. Other international organizations, governmental and non-governmental, in liaison with ISO and IEC, also take part in the work. In the field of information technology, ISO and IEC have established a joint technical committee, ISO/IEC JTC 1.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

The main task of the joint technical committee is to prepare International Standards. Draft International Standards adopted by the joint technical committee are circulated to national bodies for voting. Publication as an International Standard requires approval by at least 75 % of the national bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this International Standard may be the subject of patent rights. ISO and IEC shall not be held responsible for identifying any or all such patent rights.

ISO/IEC 13568 was prepared by Joint Technical Committee ISO/IEC JTC 1, *Information technology*, Subcommittee SC 22, *Programming languages, their environments and system software interfaces*.

Annexes A to C form a normative part of this International Standard. Annexes D and E are for information only.

Introduction

This International Standard specifies the syntax, type system and semantics of the Z notation, as used in formal specification.

A specification of a system should aid understanding of that system, assisting development and maintenance of the system. Specifications need express only abstract properties, unlike implementations such as detailed algorithms, physical circuits, etc. Specifications may be loose, allowing refinement to many different implementations. Such abstract and loose specifications can be written in Z notation.

A specification written in Z notation models the specified system: it names the components of the system and expresses the constraints between those components. The meaning of a Z specification—its semantics—is defined as the set of interpretations (values for the named components) that are consistent with the constraints.

Z uses mathematical notation, hence specifications written in Z are said to be formal: the meaning is captured by the form of the mathematics used, independent of the names chosen. This formal basis enables mathematical reasoning, and hence proofs that desired properties are consequences of the specification. The soundness of inference rules used in such reasoning should be proven relative to the semantics of the Z notation.

This International Standard establishes precise syntax and semantics for a system of notation for mathematics, providing a basis on which further mathematics can be formalized.

Particular characteristics of Z include:

- its extensible toolkit of mathematical notation;
- its schema notation for specifying structures in the system and for structuring the specification itself; and
- its decidable type system, which allows some well-formedness checks on a specification to be performed automatically.

Examples of the kinds of systems that have been specified in Z include:

- safety critical systems, such as railway signalling, medical devices, and nuclear power systems;
- security systems, such as transaction processing systems, and communications; and
- general systems, such as programming languages and floating point processors.

Standard Z will also be appropriate for use in:

- formalizing the semantics of other notations, especially in standards documents.

This is the first ISO standard for the Z notation. Much has already been published about Z. Most uses of the Z notation have been based on the examples in the book “Specification Case Studies” edited by Hayes [2][3]. Early definitions of the notation were made by Sufrin [14] and by King *et al* [8]. Spivey’s doctoral thesis showed that the semantics of the notation could be defined in terms of sets of models in ZF set theory [11]. His book “The Z Notation—A Reference Manual” [12][13] is the most complete definition of the notation, prior to this International Standard. Differences between Z as defined here and as defined in [13] are discussed in [15]. This International Standard addresses issues that have been resolved in different ways by different users, and hence encourages interchange of specifications between diverse tools. It also aims to be a complete formal definition of Z.

Information technology— Z formal specification notation— Syntax, type system and semantics

1 Scope

The following are within the scope of this International Standard:

- the syntax of the Z notation;
- the type system of the Z notation;
- the semantics of the Z notation;
- a toolkit of widely used mathematical operators;
- L^AT_EX [10] and e-mail mark-ups of the Z notation.

The following are outside the scope of this International Standard:

- any method of using Z, though an informative annex (E) describes one widely-used convention.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this International Standard. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO/IEC 10646-1:2000, *Information technology—Universal Multiple-Octet Coded Character Set (UCS)—Part 1: Architecture and Basic Multilingual Plane, with amendment 1, with its amendments and corrigenda*

ISO/IEC 10646-2:2001, *Information technology—Universal Multiple-Octet Coded Character Set (UCS)—Part 2: Supplementary Planes*

ISO/IEC 14977:1996, *Information technology—Syntactic metalanguage—Extended BNF*